# EXCERPT FROM

## Chaos In Boxes

twisted adventures in music theory

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# CHAPTER Φ

## The Times-Table Mirror

*Penny universities...*

Saturated in awe of the intricate patterning so prevalent in the world around me and faced with the luxury of exploring these realms, I found myself once again situated amongst the comfort of random human noise at another downtown coffee-shop, sipping tea with my calculator and notepad for hours on end.

Entwined in the fibres of ponderance and discovery, my mission on this occasion was to hunt down any musical treasures to be found in the times-tables.

First, I wrote out all 9 of the times-tables with the answers reduced to a single digit. The following example shows the 4x table:

```
4 x 1 = 4       SR = 4
4 x 2 = 8       SR = 8
4 x 3 = 12      SR = 1 + 2 = 3
4 x 4 = 16      SR = 1 + 6 = 7
4 x 5 = 20      SR = 2 + 0 = 2
4 x 6 = 24      SR = 2 + 4 = 6
4 x 7 = 28      SR = 2 + 8 = 10, 1 + 0 = 1
4 x 8 = 32      SR = 3 + 2 = 5
4 x 9 = 36      SR = 3 + 6 = 9
4 x 10 = 40     SR = 4 + 0 = 4
...ad infinitum
```

A repeating pattern of 4 8 3 7 2 6 1 5 9 was revealed by the 4x table. The rest of the times-tables gave the following repeating patterns:

```
1x:   1 2 3 4 5 6 7 8 9' 1 2 3 4 5 6 7 8 9' ...
2x:   2 4 6 8 1 3 5 7 9' 2 4 6 8 1 3 5 7 9' ...
3x:   3 6 9' 3 6 9' 3 6 9' 3 6 9' 3 6 9' 3 6 9' ...
4x:   4 8 3 7 2 6 1 5 9' 4 8 3 7 2 6 1 5 9' ...
5x:   5 1 6 2 7 3 8 4 9' 5 1 6 2 7 3 8 4 9' ...
6x:   6 3 9' 6 3 9' 6 3 9' 6 3 9' 6 3 9' 6 3 9' ...
7x:   7 5 3 1 8 6 4 2 9' 7 5 3 1 8 6 4 2 9' ...
8x:   8 7 6 5 4 3 2 1 9' 8 7 6 5 4 3 2 1 9' ...
9x:   9' 9' 9' 9' 9' 9' 9' 9' 9' 9' 9' 9' 9' 9' 9' 9' 9' 9' ...
```

In a zippy flash of eureka-gasm brainfartness I thought, "I will

arrange each of these patterns into its own circular graph. I shall connect the dots!"

And so I connected the dots. I had imagined that some sort of nail-art type of imagery would appear, which it did. But some unexpected twists and turns emerged as well.

Firstly, I arranged the numbers 1 thru 9 clockwise around the outside of a circle, establishing a template for the graphs:



Next, I connected the dots around the circle in the skein order for each times-table. Using the 4x table as an example again (with the sequence 4 8 3 7 2 6 1 5 9 as shown on the previous page):
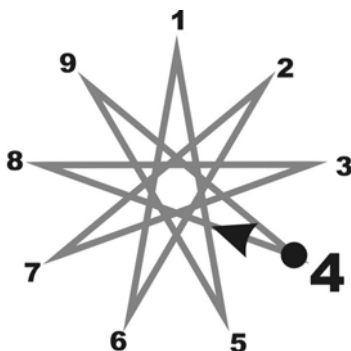


Cool looking pattern , eh?

But the real big surprise is the symetry in the *sequence of shapes* that are created through this procedure. Not including the 9x table, which seems to be reserved for its own special function as a sort of "mirror" or "grounding anchor" for the rest of the system, the series of 8 shapes *evolves palindromically.* They appear in the same order, backwards or forwards.

Also, polarity in regards to clockwise or counter-clockwise is reversed across the mirror. The 4x table shown above, for example, spins clockwise whereas its twin, the 5x table, is *counter*-clockwise.

The following diagram shows the pattern of patterns:
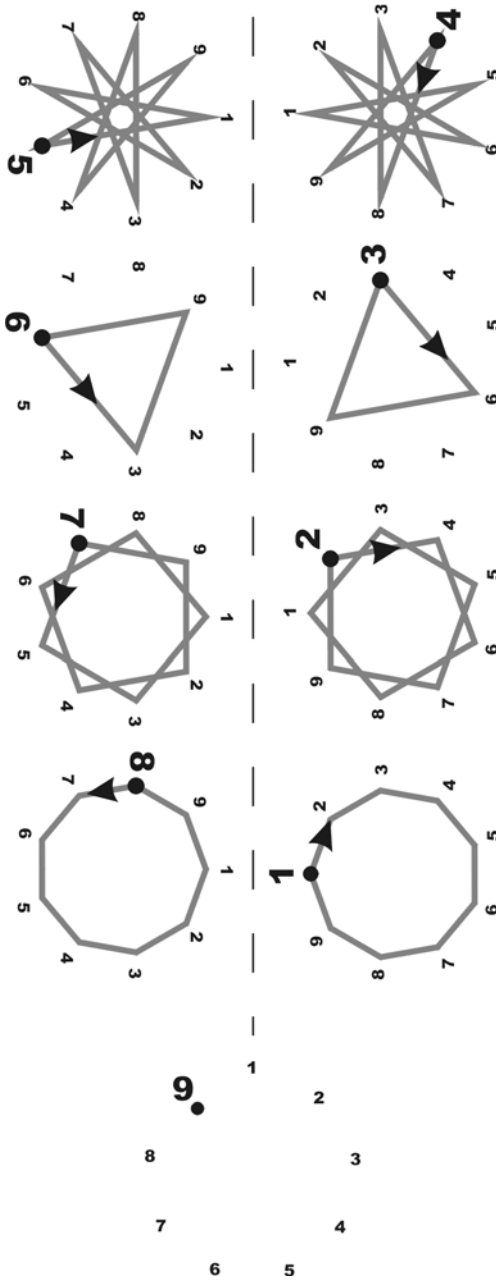
    I originally had the diagram laid out lengthwise with all the shapes side-by-side, but I wanted it to fit on a single page without being too small to read.  The folded arrangement has the added bonus of resembling a V-8 engine!  The empty 9x graph at the bottom of the previous diagram resembles the engine's *distributor,* with the dot at 9 perhaps corresponding to the electrical supply wire.  I wonder - what sort of pattern would emerge if the shapes were re-arranged to conform to the firing order of the sparkplugs of a real V-8?

    Another idea for the deferral list, I suppose.  Second edition... "Chaos In Boxes volume 2,"perhaps?

    [There's this little yellow spider that keeps crawling on the computer screen; when the typing cursor moves towards the spider, it jumps out of the way every time, to avoid being attacked by the cursor.  Silly little spider.]

    The pattern gains new beauty and grace when arranged as a circle of circles:
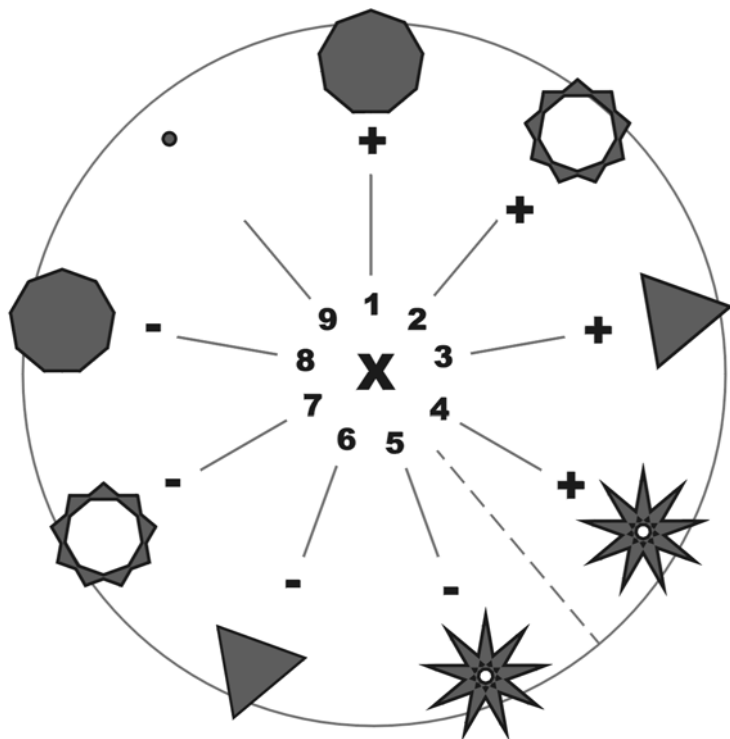


# The Times-Table Mirror

    *Eureka!*

Eventually I noticed that, by folding the circle in half along the dotted line, the numbers match up a certain specific way: the nonagons are numbered 1 and 8; the flowers are 2 and 7; the triangles are 3 and 6; the spikeys are 4 and 5. Each number pair totals 9, which is also the mirror number of the entire system - what an incredible coincidence!

Even more amazingly and perhaps enough justification alone for the mere existence of this entire book, this "folding 9-sum" property is perfectly echoed in the bi-directional measurement of musical intervals and their sum of 9: according to standard Western music theory, a movement a *third upwards* is equivalent to moving a *sixth downwards* - and, of course, three plus six equals *nine*.

Also noteworthy is the flip-flop pairing of the major/minor quality of these interval additions. That is, an upwards *minor* third (eg. E to G) equals a downward *major* sixth (eg. E to G), whereas an upwards *major* third (eg. E to G#) equals a downward *minor* sixth (eg. E to G#). Comparison of this musical fact to the polarities in clockwise-ness found in each pair of times-table shapes confirms yet another new point of analogy perhaps significant enough for contemplation.

Number nine.
Number nine.
Number nine.
Number nine.
Number nine.
Number nine.
Number nine.
Number nine.
Number nine.

# CHAPTER Φ

## The Modal Mirror (Bumblebees & Basketballs)

Let's flip some scales *upside-down!*

Recall from the beginning of this Section that a half-step is one fret's distance on the guitar, and a whole-step is two frets (W = whole-step, H = half-step). Starting from its root note and moving upwards one note at a time, that old cliché the Major Scale has a stepwise spelling of W  W  H  W  W  W  H.

Its interval spelling: root, major second, major third, perfect fourth, perfect fifth, major sixth, major seventh. This can be abbreviated R, 2, 3, 4, 5, 6, 7; other scales would likely have modifications to this, indicated as "flat 2" or "minor 6" or "m7" or "sharp 4" or "#5" or other alterations.

With no sharps or flats, the C Major Scale is the simplest example to discuss in terms of note names: C plus a whole-step equals D, and D plus a whole-step is E, and E plus a half-step is F, and F plus a whole-step is G, and G plus a whole-step is A, and A plus a whole step is B, and B plus a half-step returns to C, completing the circle.

*Well...* how about starting at C and *minusing* the same pattern of steps, spelling *downwards* instead of upwards? C *minus* a whole-step equals B-flat. There is no B-flat in the key of C major... already apparent, a new scale is going to be generated that is different from the C Major Scale... a mystery unfolding!

Continuing the process:

$$C - W = B\text{-flat}$$
$$B\text{-flat} - W = A\text{-flat}$$
$$A\text{-flat} - H = G$$
$$G - W = F$$
$$F - W = E\text{-flat}$$
$$E\text{-flat} - W = D\text{-flat}$$
$$D\text{-flat} - H = C$$

This new collection of notes is commonly known as the Phrygian mode, and has a rather flamenco-ish sort of sound.

*"Mode?"* What the heck is that? Fridge... Ice cream? Flamenco ripple? Huh?"

Modes are simply phase-evolutions of an existing "parent" scale, much like phases of the moon. A parent scale with a certain number of notes has the same number of "child" modes, each of which is also a musical scale. Every note gets a chance to be the root of its own mode.

Each of these modes exudes its own unique "mood" which differs from its parent scale as well as its sibling modes.  The child and parent modes basically have an equivalent level of expressive sovereignty, to the extent that any of the modes could rightly be considered the "original from which the others are derived".  However, one of them is usually designated as the originator.

"Mode 1" refers to the perceived originator.  Therefore, mode 1 of the Major Scale (W W H W W W H) is simply *Do-Re-Mi-Fa-Sol-La-Ti-Do* itself.  By definition, mode 2 starts on note 2 of mode 1 and has a shifted version of mode 1's step-pattern: W H W W W H W.  Mode 3 begins on the third note of mode 1: H W W W H W W.  This derivation process continues through all 7 possibilities:

```
mode 1:*           W W H W W W H
mode 2:**            W H W W W H W
mode 3:***            H W W W H W W
mode 4:****             W W W H W W H
mode 5:*****             W W H W W H W
mode 6:******             W H W W H W W
mode 7:*******             H W W H W W W
```

Each mode sounds different because its step pattern is unique, and therefore so is its collection of intervals.  Through music, numbers exhibit resonance, colour and emotion.  Mode vocabulary is extremely useful for composers and improvisers alike!

Although any conceivable scale is automatically a mode generator, when a musician talks of "modes" these days they're usually referring to the 7 modes of the Major Scale, listed in this page's footnotes.

*So...*

That's enough background theory for now; back to the process of flipping scales upside down!  When Ionian is inverted, Phrygian mode is the result.  An intriguing transformation indeed!

I went ahead and flipped each of the 7 modes to establish their "undertone" counterparts.  The results amazed me, especially when plotted on a circular graph.  With the mode numbers evenly spaced sequentially around a circle, the flipperoos make a bunch of parallel lines going back and forth across the circle.

―――――――――――――――――――――――――――――――――――――――

* a.k.a. Ionian:            R, 2, 3, 4, 5, 6, 7.
** a.k.a. Dorian:           R, 2, m3, 4, 5, 6, m7.
*** a.k.a. Phrygian:        R, m2, m3, 4, 5, m6, m7.
**** a.k.a. Lydian:         R, 2, 3, sharp 4, 5, 6, 7.
***** a.k.a. Mixolydian:    R, 2, 3, 4, 5, 6, m7.
****** a.k.a. Aeolian:      R, 2, m3, 4, 5, m6, m7.
******* a.k.a. Locrian:     R, m2, m3, 4, flat 5, m6, m7.

The arrangement resembles a bumblebee, or maybe a basketball:
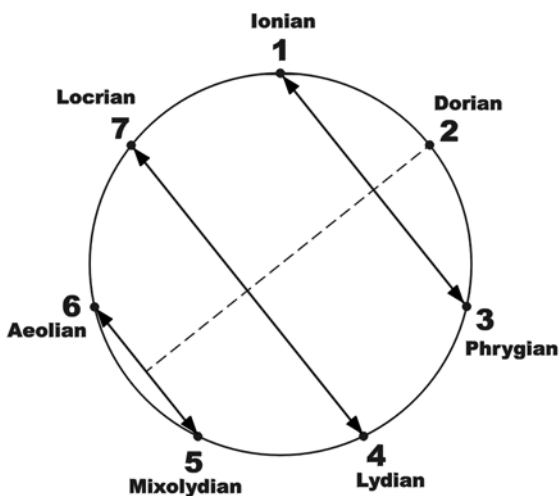


**The Modal Mirror**

Curiously, the Dorian* mode remains the same when it's flipped upside-down!  It also presides at the exact center of the system, at one end of the perpendicular to the parallels;** mode 2*** is the mirror axis between undertone and overtone, the glassy surface of the 7 tonal seas!  With some practical experimentation, the expressive potential of jumping across the Modal Mirror becomes self-evident.

And so I pondered...   what sort of results would come from inverting some other scale types?  What about inverting the Blues scale, Harmonic Minor, Jazz Minor, Arabian, Lydian Dominant, Hungarian Gypsy, Enigmatic, Balinese, Algerian, Hirajoshi, Pelog, Hindustan, Ritusen, Piongio, Prometheus, Scriabin, Iwato, Egyptian,

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\* The modes were named after tribes of people in and around ancient Greece; the complete history of modes, and their technical specifications, is tangled and inconsistent, so I have only presented the "modern modes" here, and avoided the old "church modes" and the original (older) Greek modes altogether.

\*\* The other end of the Dorian mirror is flanked by Aeolian and Mixolydian - each differing from Dorian by only one interval, further illustrating Dorian's "in-between-ness" with beautiful consistency.

\*\*\* Notice how, in Chapter Φ, "The Times-Tables Mirror," the number *9* is the axis of the system.  In the Modal Mirrors, *2* is the axis.  It just so happens that degree *2* of a typical 7-note scale is also referred to as *9*, since 8 is the octave which is a duplicate of the root: 8=1, 9=2, 10=3, 11=4, 12=5, 13=6.  *A direct connection between the times-tables and the 7 modes...* who knew?  See Appendix 1497, "Palindromic Skein Sequence In The Squares" for more intriguing 9-axis stuff.  ($0^2 = 9$?)

Romanian Minor, Kumoi, Jewish, Mongolian, Locrian Major, and any others from the endless palette of scales?

By digging into the undertone side of things, the possibilities are suddenly *twice as endless* as they already were!  [2 x ∞ = ∞.]

Exploring all of these inverted scales on the guitar could provide years of entertainment and study, if not lifetimes!  Are we there yet? A short poem to celebrate the notion:

<div align="center">

Alas
to be a vampire
and   never   age or
die
!

</div>

Gotta start somewhere - how about taking a step towards simplicity by using only 5 notes instead of 7... What happens when the classic Minor Pentatonic scale (W+H  W  W  W+H  W) is inverted?

<div align="center">

| A Minor Pentatonic | = A C D E G A |
| upside-down A Minor Pentatonic | = A F# E D B A |

</div>

A musical composition could feasibly concentrate on the upside-down Minor Pentatonic tonality for one section and then flip to the traditional upward spelling for a melodic contrast.  Or, both at the same time!  The above-water spelling could be manifested by a flute while the bass carries the inverted contour, for example.

Q: "What happens when *both sides* of the Minor Pentatonic mirror are combined into *one scale?"*

A: Collecting all notes from both of these 5-note scales, a new scale with 7 notes (A  B  C  D  E  F#  G) is produced - a more relevant answer than I was expecting, as these are the same notes as A Dorian - the exact same mode that functions as mirror-axis of the amazing Modal Mirror circle!

<div align="center">

Minor Pentatonic scale
+
inverted Minor Pentatonic scale
=
Dorian, axis mode of the Modal Mirror

</div>

Go figure!*

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* I'd already been using the modes on the guitar for over 15 years before discovering these amazing symmetries - right under my nose all this time!